Optimization of Multiple Exposure Gratings for Widely Tunable Lasers

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***Abstract*—A theoretical study of the reflection spectra of multiple holographic exposure distributed Bragg reflector gratings is presented. We will show that such a grating can exhibit a uniform reflection comb if the relative phases of the superimposed gratings are well chosen. The tolerances on the optimal phases are of the order of 0.09 radians for six superimposed gratings to 0.04 radians for ten gratings.**

***Index Terms*—Distributed Bragg reflector lasers, holographic gratings, laser tuning, optical waveguide filters, semiconductor lasers, wavelength-division multiplexing.**

I. Introduction

WIDELY tunable lasers are promising components for future HD-WDM networks. Most of the present widely tunable lasers employ waveguide-grating reflectors with a comb-like reflectivity spectrum [1]–[6]. Ideally, these reflectors have a number of uniform reflection peaks within a limited wavelength range. In the case of a sampled grating (SG) [1] the peak-reflectivity decreases with the distance from the central wavelength and long gratings are necessary to obtain strong reflection. On the other hand, the SG has the advantage that only a single holographic exposure is needed during processing. Super structure gratings (SSG) [2] have been designed with highly uniform reflection peaks, but these need very precise e-beam lithography for the definition of the (complex) grating pattern. Recently, the concept of the SSG has been improved by the binary superimposed gratings (BSG), which can generate similar reflection spectra, but require much less precision in the e-beam lithography [5].

In this letter, we will study the characteristics of multiple exposure Bragg gratings. These can be seen as an extension of ordinary Bragg gratings toward multiwavelength reflectors, however still relying on holographic exposure for the fabrication. A. Talneau *et al.* [4], [6] first presented these, but until now no theoretical calculations of reflection spectra have been presented. We will show that these multiwavelength gratings (MWG) can generate uniform reflection combs, if the relative phases of the superimposed Bragg gratings are suitably chosen. In reported experiments, e.g., [6], these phases were purely left to chance.

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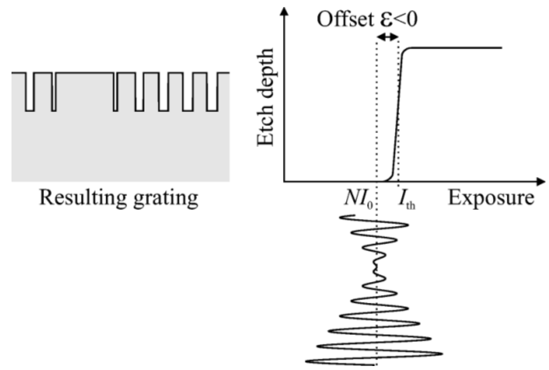

Fig. 1. Transfer of exposure intensity (integrated over time) to etch depth. Example of holographic exposure distribution for superimposed Bragg gratings and resulting grating.

II. Theory

To fabricate a MWG, a thin photoresist layer is sequentially exposed by two-beam interference patterns with different pitches [4], [6]. Because of the thresholding effect of the photoresist development and the characteristics of the etching process, the transfer of the superimposed fringe patterns into semiconductor material is highly nonlinear. Especially when using dry-etching techniques, the resulting grating has a more or less rectangular profile. This means that the transfer of exposure intensity (integrated over time) to etch depth can be approximated by a step function (Fig. 1).

If the different exposure steps have equal strength (product of exposure time and average intensity is equal), the total exposure distribution as function of the position z along the grating can be described by

$$I(z) = \frac{I_0}{2} \sum_k \left\{ 1 + \cos\left(\frac{2\pi z}{\Lambda_k} + \phi_k \right) \right\}$$

where Λ_k is the pitch and ϕ_k is the phase of grating k at a certain reference position $z = 0$.

We will limit our study to those cases with an even number of superimposed gratings $2N$ (analogous results can be found for odd numbers), with uniformly spaced frequencies centered on F_0, so that the total exposure distribution can be written as

$$I(z) = NI_0 + I_0 \sum_{k=1}^{N} \left\{ \begin{array}{c} \cos(2\pi \Delta F z(k - \frac{1}{2}) + \frac{1}{2}(\phi_k - \phi_{-k})) \\ \cdot \cos(2\pi F_0 z + \frac{1}{2}(\phi_k + \phi_{-k})) \end{array} \right\}.$$

If the gratings with frequencies F_k and F_{-k} symmetric versus the central frequency F_0 have opposite phases at $z =$

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TABLE I
NUMERICAL VALUES OF CALCULATION PARAMETERS

Parameter	Value	Unit
Grating center frequency F_0	4.2562	μm^{-1}
Grating frequency spacing ΔF	$5.4346\ 10^{-3}$	μm^{-1}
Effective refractive index n	3.2985	
Central wavelength λ_0	1.55	μm
Coupling coefficient for $2N = 6$	$\kappa = 2(n_1\text{-}n_2)/\lambda_0 = 3.000\ 10^{-3}$	μm^{-1}
Coupling coefficient for $2N = 10$	$\kappa = 2(n_1\text{-}n_2)/\lambda_0 = 3.871\ 10^{-3}$	μm^{-1}
Weights w_k, $2N = 6$ (symmetrical)	... 0 1 3 5 10 10 10 ...	
Weights w_k, $2N = 10$ (symmetrical)	... 0 1 2 3 4 5 10 10 10 10 10 ...	

$0, \phi_k = -\phi_{-k}$, this can be simplified to

$$I(z) = NI_0 + I_0 \cos(2\pi F_0 z) \cdot E(z)$$

with

$$E(z) = \sum_{k=1}^{N} \cos(2\pi \Delta F z(k - \tfrac{1}{2}) + \phi_k).$$

We define the offset ε as the difference between the average exposure NI_0 and the threshold I_{th} (Fig. 1). In general, the result is a grating with continuously varying duty-cycle, including regions where everything is etched away ($\varepsilon > 0$, overexposure) or where there is no etching ($\varepsilon < 0$, underexposure).

To calculate the reflection spectra of such gratings, we used a transfer-matrix method [7]. The reflection spectrum is the same as that of a plane wave reflecting off a layered structure perpendicular to the propagation axis, with alternating layers with refractive indices n_1 and n_2, which corresponds to a grating coupling coefficient $\kappa = 2(n_1 - n_2)/\lambda_0 = \pi \Delta n_I / \lambda_0$. Here, $\Delta n_I = 2/\pi\ (n_1 - n_2)$ is the first order Fourier-component of a square wave with amplitude $n_1 - n_2$. The calculation of the reflection spectrum generally involves the multiplication (for each wavelength) of several thousands of transfer-matrices, as the alternating layers have a continuously varying thickness. In the special case with anti-symmetric phases $\phi_k = -\phi_{-k}$ and offset $\varepsilon = 0$ however, nearly all layers have the same thickness and therefore the long sequence of transfer matrices can be regrouped analytically into a sequence of only a few tens of matrices, which greatly reduces computation time.

III. CALCULATION EXAMPLES

The simplest example of a multiwavelength grating is the Moiré grating, where $2N = 2$. Fig. 2(a) shows the reflection spectrum.[1] For $2N > 2$ the relative phases ϕ_k begin to play a role. Compare for example the Fig. 2(b), (c), and (d) where $2N = 6$. In Fig. 2(b) all phases are 0, while in Fig. 2(c) we have made $\phi_3 = -\phi_{-3} = \pi$. Fig. 2(d) shows the reflection spectrum for a set of arbitrarily chosen phases. If the aim is to obtain $2N$ equally strong reflection peaks at the wavelengths corresponding to the different holographic pitches Λ_k, the spectrum of Fig. 2(c) is clearly much better than that of Fig. 2(b) and (d).

[1] Numerical values used for different calculation parameters are given in Tables I and II.

TABLE II
OPTIMIZED PHASES FOR SEVERAL VALUES OF THE NUMBER OF PEAKS $2N$

$2N$	ϕ_{10}	ϕ_{20}	ϕ_{30}	ϕ_{40}	ϕ_{50}	ϕ_{60}
2	0					
4	0	-1.844				
6	0	-0.350	-2.542			
8	0	-2.646	-0.981	-2.943		
10	0	-1.713	-0.148	-0.069	+3.100	
12	0	-1.351	2.276	-0.943	+0.974	+1.169

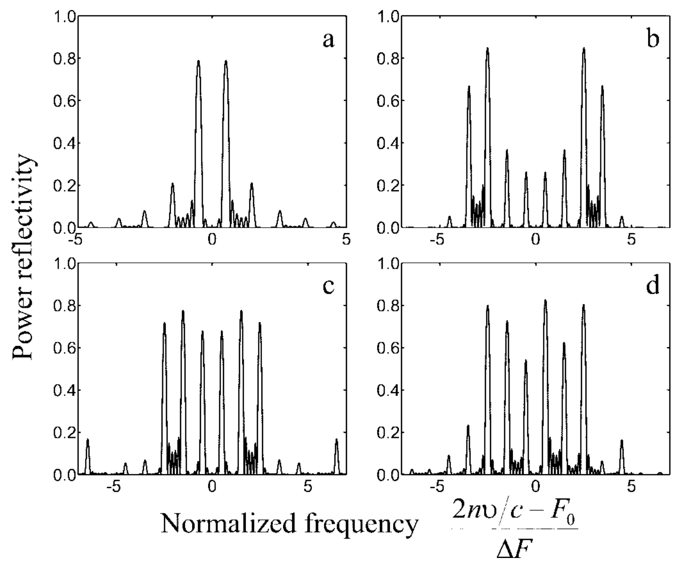

Fig. 2. Examples of reflection spectra of 2 (a) or 6 (b, c, d) superimposed Bragg gratings. (a) $[\phi_1 \phi_{-1}] = [00]$, (b) $[\phi_3 \cdots \phi_{-3}] = [0\ 0\ 0\ 0\ 0\ 0]$, (c) $\cdots = [-\pi\ 0\ 0\ 0\ 0\ \pi]$, (d) $\cdots = [2.83\ -1.68\ 0.67\ -0.09\ 2.46\ 1.65]$.

Another important parameter is the grating length L. The absolute value of the envelope function $E(z)$ is periodic with a (super)period $1/\Delta F$. For the reflection peaks to be sufficiently narrow and for the spectrum to be more or less independent of the position of the grating edge within the superperiod, the total length L should be at least a few (e.g., 5) times this period. In our calculations we took $L = 6/\Delta F$.

IV. OPTIMIZATION

Here, we will discuss the optimization of the reflection spectrum as function of the phases ϕ_k, where the objective is to obtain equally strong reflection peaks at the wavelengths corresponding to the different holographic pitches Λ_k, and weak reflection at other wavelengths. The optimization was done for gratings with antisymmetric phases $\phi_k = -\phi_{-k}$ and offset $\varepsilon = 0$, because of the reduced computation time and more importantly because only these gratings have symmetric reflection spectra. We made $\phi_1 = -\phi_{-1} = 0$, as adding a value of $(2k-1)\theta$ to ϕ_k ($k \geq 1, \theta$ any arbitrary angle) and maintaining the antisymmetry, would give the same grating, only shifted over a distance $\Delta z = \theta/(\pi \Delta F)$.

As cost function we took a weighted sum of squared errors between the peak reflectivities $R_{\pm k}$ at the frequencies

$$\upsilon_{\pm k} = \frac{c}{2n}\left[F_0 \pm \left(k - \frac{1}{2}\right)\Delta F\right]$$

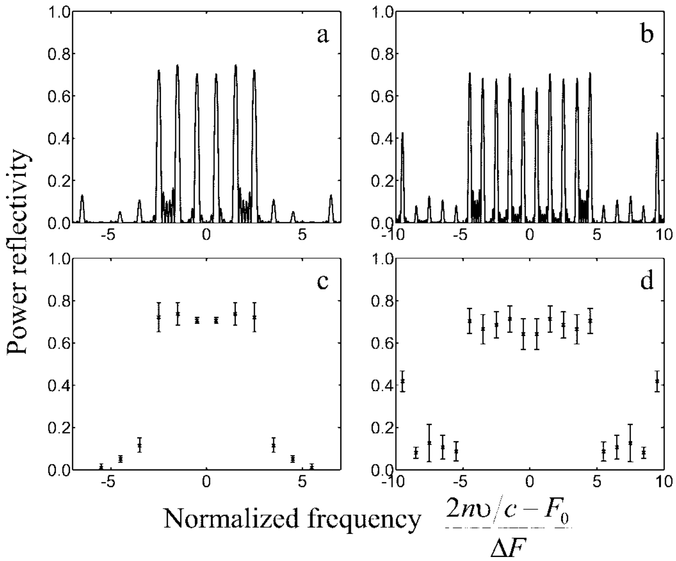

Fig. 3. Optimized reflection spectra for (a) 6 and (b) 10 superimposed Bragg gratings. Error bars (95% probability intervals) for Gaussian errors on the relative phases with a standard deviation (c) $\sigma = 2\pi/100$ resp. (d) $\sigma = 2\pi/200$.

and $R_{\max}\delta_{\pm k}$, where $R_{\max}$ is the overall maximum reflectivity, $\delta_{\pm k} = 1$ for $1 \leq k \leq N$ and $\delta_{\pm k} = 0$ for all other k

$$\text{cost} = \sum_k \{w_k(R_{\max}\delta_k - R_k)^2 + w_{-k}(R_{\max}\delta_{-k} - R_{-k})^2\}.$$

We minimized this function using the Nelder–Mead simplex search algorithm [8], with randomized starting values for $\phi_2, \cdots, \phi_N$ to make sure that we find a global minimum.

Fig. 3(a) and (b) show the optimized reflection spectra for $2N = 6$ and $2N = 10$. In order to estimate the tolerances on the optimum phases ϕ_{k0}, we calculated multiple reflection spectra when Gaussian errors with a given standard deviation σ were added to the optimum phases. Fig. 3(c) and (d) show the intervals that contain 95% of the calculated peak reflectivities for the standard deviations $\sigma = 2\pi/100$ for $2N = 6$ and $\sigma = 2\pi/200$ for $2N = 10$. In both cases the maximum error is approximately 10% of the maximum reflectivity $R_{\max}$. As could be expected, the tolerances on the phases ϕ_{k0} become tighter as the number of peaks $2N$ increases: the tolerances for a maximum reflectivity deviation of $0.1R_{\max}$ are of the order of 0.09 radians for six peaks, decreasing to 0.04 radians for ten peaks (a 95% probability corresponds to a maximum phase deviation of approximately 1.4σ).

Finally, we investigated the influence of a nonzero offset ε on the reflection. It appears that the peak-reflectivities decrease, which is caused by the decrease of the effective coupling coefficient for an average grating duty-cycle different from 50%. Also, the reflectivities for the longer (shorter) wavelengths are slightly stronger for positive (negative) ε, and the parasitic reflection peaks are strongly suppressed.

V. Conclusion

The reflection spectra of multiwavelength gratings (MWG's) have been studied theoretically. Some general design rules were derived: the grating length L should be at least a few times the "superperiod" $1/\Delta F$, and the relative phases ϕ_k of the superimposed Bragg gratings should be carefully controlled. Numerical optimization of the relative phases ϕ_k resulted in uniform reflection combs with peaks at all the corresponding Bragg wavelengths. Tolerances on the optimized phases ϕ_{k0} are of the order of 0.09 radians for six peaks to 0.04 radians for 10 peaks.

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