Nonlinear photonic crystals near the supercollimation point

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We uncover a strong coupling between nonlinearity and diffraction in a photonic crystal at the supercollimation point. We show that this is modeled by a *nonlinear diffraction term* in a nonlinear-Schrödinger-type equation in which the properties of solitons are investigated. Linear stability analysis shows solitons are stable in an existence domain that obeys the Vakhitov–Kolokolov criterium. In addition, we investigate the influence of the nonlinear diffraction on soliton collision scenarios. © 2008 Optical Society of America

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Photonic crystals (PhCs) are under active investigation owing to rich physics and all-optical signal control [1–3]. One of the striking features in PhCs is the supercollimation (SC) effect, which was experimentally [4,5] and theoretically [6–8] examined. This effect originates from the possibility of obtaining flat regions in the spatial dispersion relation or equifrequency contours of PhCs. At these particular points the phase propagation components along the direction of a beam are equal. In this way all components of the beam travel with the same phase velocity, and it becomes nondiffractive. Recently, centimeter-scale SC was achieved in a large-area two-dimensional PhC [9]. However, the study of nonlinear effects around these SC points is largely unaddressed. Here we propose a more fundamental approach, as opposed to a more numerical one [10].

In this Letter we derive a phenomenological model that describes light beam propagation in nonlinear PhCs around the SC point. In most optical systems nonlinearity is a small perturbation and hence is added to the linear equation of motion. In contrast, in the current system one uncovers a strong coupling between nonlinearity and diffraction. More specifically, a nonlinear diffraction term is introduced into the nonlinear Schrödinger (NLS) equation. This term can control the magnitude and sometimes even the sign of the diffraction. As a result, this is a unique system, and new physical phenomena emerge. For example, this modified equation gives rise to solitons up to an upper threshold for the propagation constant. Linear stability analysis demonstrates that the solitons are stable in the existence domain and obey the Vakhitov–Kolokolov criterium. In addition, we examine soliton interaction scenarios.

To deduce the model equations we consider a 3D PhC with the same periodicity in the transversal x and y directions (a scheme shown in Fig. 1) and write the electric field of a beam in the PhC as $\mathbf{E}(x,y,z,t) = \mathbf{F}(x,y,z)A(x,y,z)\exp(ik_z^c z - i\omega t)$, with k_z^c being the propagation constant of the central Bloch mode, ω being the frequency of the beam, $A(x,y,z)$ being the slowly varying amplitude, and $\mathbf{F}(x,y,z)$ being the Bloch mode profile corresponding to ω and k_z^c. The propagation direction is along z.

We study beams in the neighborhood of an SC point, so $\omega \approx \omega_{SC}$ with ω_{SC} being the SC frequency. The special dispersion relation around this point is shown in Fig. 1. The sign and strength of diffraction depends strongly on the position of ω versus ω_{SC}. Exactly at the frequency ω_{SC}, all components of the beam travel with the same propagation constant along z, noted by k_z^{SC}, and there is no diffraction. For frequencies different than ω_{SC} we approximate the equifrequency contours by a parabola. In this case diffraction can be positive or negative, depending on

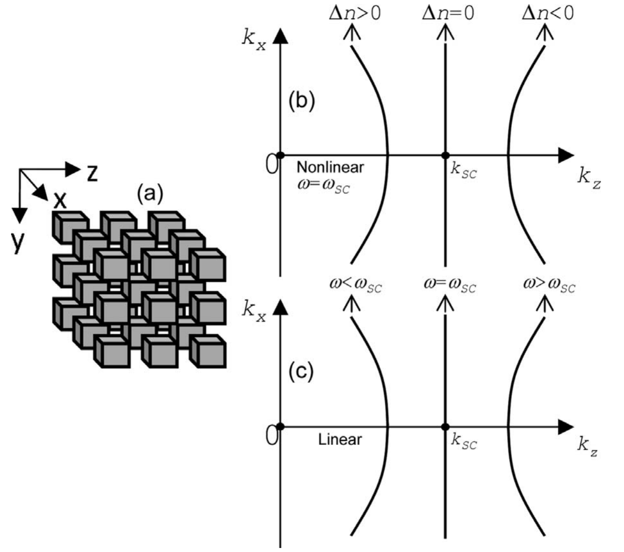

Fig. 1. (a) Scheme of a 3D PhC. (b) Depiction of the linear dispersion relation in the proximity of a SC point. (c) Nonlinearity gives rise to an index change δn and is modeled by a shift of the dispersion relation, here shown for the particular input frequency $\omega = \omega_{SC}$.

the position of ω versus ω_{SC}. Furthermore, the strength of diffraction increases as the difference between ω and ω_{SC} increases.

To include nonlinearity we apply first-order perturbation theory [11]. Thus, the nonlinear interaction causes a small shift of the local dispersion relation (Fig. 1), which is equivalent to shifting ω_{SC}. To first order this shift is given by

$$\frac{\Delta\omega}{\omega_{SC}} = -\frac{1}{3}\frac{\int \mathrm{d}\mathbf{r} n_2(\mathbf{r}) n(\mathbf{r})[(\mathbf{F}\cdot\mathbf{F})(\mathbf{F}^*\cdot\mathbf{F}^*) + 2|\mathbf{F}|^4]}{\int \mathrm{d}\mathbf{r} n^2(\mathbf{r})|\mathbf{F}|^2} \times |A(x,y,z)|^2 \equiv \kappa|A(x,y,z)|^2, \quad (1)$$

with κ being a nonlinear coefficient calculated from the linear Bloch mode profile.

The nonlinearity shifts ω_{SC} to $\omega_{SC}+\Delta\omega$, so by Taylor expanding the dispersion relation around the SC point the modified dispersion relation becomes approximately

$$k_z - k_z^{SC} + \alpha_1(\omega - \omega_{SC} - \kappa|A|^2\omega_{SC}) = \beta_1(\omega - \omega_{SC} - \kappa|A|^2\omega_{SC})(k_x^2 + k_y^2), \quad (2)$$

with k_z being the longitudinal and k_x, k_y being the transverse propagation vector components. The term with α_1 corresponds to the frequency dependence of the central k_z component (thus at k_x, $k_y=0$). Similarly, the term with β_1 describes the frequency change of the curvature (or the diffraction). Note that we neglect higher-order diffraction terms, such as fourth-order diffraction, as we are considering propagation of a broad beam with respect to the PhC period. We transform Eq. (2) into the space domain and arrive at the following equation in dimensionless form: $i(\partial q/\partial\xi) + 1/2\,\alpha\nabla^2 q - 1/2\,\beta|q|^2\nabla^2 q + \gamma|q|^2 q = 0$, where $\nabla^2 = (\partial^2/\partial\eta^2) + (\partial^2/\partial\zeta^2)$, with the transverse coordinates η and ζ scaled to the spatial characteristic width W_0 and ξ being the longitudinal coordinate scaled to the free-space diffraction length $L_d = 2\pi W_0^2/\lambda$, for Gaussian-like beams [12]. $\alpha = -2\beta_1(\omega - \omega_{SC})L_d/W_0^2$ indicates the linear diffraction strength, and its sign characterizes the type of linear diffraction. The novel nonlinear diffraction term is preceded by $\beta = 2\beta_1\kappa\omega_{SC}c^2L_d/W_0^2$, with c being the speed of light. The usual nonlinear term has $\gamma = -\alpha_1\kappa\omega_{SC}c^2L_d$. In the following we employ $\delta n(I) > 0$ (thus $n_2 > 0$) and $\beta_1 > 0$.

In this Letter we will mainly show results concerning 2D PhCs, so the resulting equation takes the form

$$i\frac{\partial q}{\partial\xi} + \frac{1}{2}\alpha\frac{\partial^2 q}{\partial\eta^2} - \frac{1}{2}\beta|q|^2\frac{\partial^2 q}{\partial\eta^2} + \gamma|q|^2 q = 0. \quad (3)$$

Note that all the coefficients for this model can be deduced from rigorous numerical simulations [9]. Recently, an equation similar to Eq. (3) was reported as the continuous approximation of the Salerno model [13–15]. Here, in contrast, we derive the model from

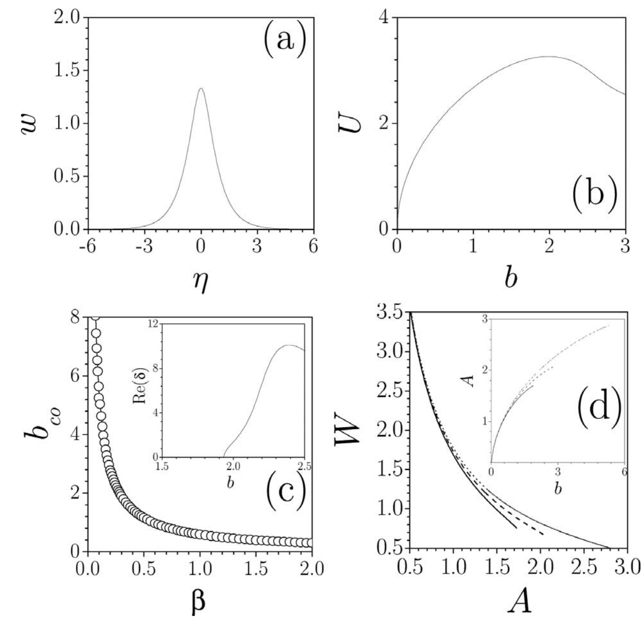

Fig. 2. (a) Profile of a soliton for $b=1$ with $\beta=0.3$. (b) The dispersion diagram for $\beta=0.3$. (c) Domain of existence for solitons in the (b,β) plane, where the inset shows the real part of the perturbation growth rate versus propagation constant for $\beta=0.3$. (d) The soliton width (FWHM) versus maximum amplitude for varying nonlinear diffraction, namely, $\beta=0.1$ (dotted curve), 0.2 (dashed curve), and 0.3 (solid curve). The inset shows the dependence of maximum amplitude on the propagation constant. $\alpha=1$ and $\gamma=1$ for all cases.

a physical system, describing light beam propagation in nonlinear PhCs with SC. Equation (3) conserves the power $U = -(1/\beta)\int \ln|\alpha - \beta|q|^2|\mathrm{d}\eta$. The stationary solutions of Eq. (3) have the form $q(\eta,\xi) = w(\eta)\exp(ib\xi)$, where a real function $w(\eta)$ and a real propagation constant b are found by iterative relaxation method. To analyze stability we examine perturbed solutions $q = (w + u + iv)\exp(ib\xi)$, where the real $u(\eta,\xi)$ and imaginary $v(\eta,\xi)$ perturbations can grow with complex rate δ upon propagation. The eigenvalue problem linearized from Eq. (3) around $w(\eta)$ is solved numerically.

One concludes that Eq. (3) allows soliton solutions [see, e.g., Fig. 2(a)], but only in a finite band of propagation constants. More specifically, there exists an upper threshold for the propagation constant (b_{co}), which depends on the strength of nonlinear diffraction β [Fig. 2(c)]. Above this value peakon solutions [13] appear, which are unphysical given the assumptions of our physical model. To model what happens beyond that point in a real PhC one would have to include more terms in Eq. (3). To emphasize, in our system it is *the nonlinear diffraction term* that leads to the reduction of the semi-infinite band of propagation constants (as in a pure cubic NLS system) to a finite one, which is typical for soliton families in models with competing nonlinearities, such as the cubic-quintic NLS equation [16]. It is noted that the power is a nonmonotonic function of the propagation constant [Fig. 2(b)]. Figure 2(d) shows the dependence of the width of solitons on the amplitude (the plot of the inset shows the dependence of the maximum amplitude on the propagation constant). As one can see from this plot, the nonlinear diffraction has a significant effect at larger amplitudes. In addition, it should be pointed out that a lower power would be needed to generate a soliton in our system, because the diffraction is so weak to start with. Thus our system may provide an experimentally favorable way to manipulate nonlinear waves.

Linear stability analysis shows that solitons are stable in the whole domain of their existence. An instability growth rate calculation is shown in the inset of Fig. 2(c). It is noted that the Vakhitov–Kolokolov criterium applies to our system for fundamental solitons, namely, solitons are stable when $\mathrm{d}U/\mathrm{d}b>0$. To confirm the outcome of the linear stability analysis we perform direct numerical simulations of Eq. (3). We employ a split-step Fourier method in combination with fourth-order Runge–Kutta to deal with the nonlinear diffraction term. The input condition is $q(\eta,\xi=0)=w(\eta)[1+\rho(\eta)]$, with $w(\eta)$ being the profile of the stationary soliton and $\rho(\eta)$ being a random noise function with a variance of up to 10%. The simulations confirm the linear stability analysis.

As another example where nonlinear diffraction affects fundamental phenomena, we report its influence on soliton collisions for different input conditions. We use as the input a soliton of the pure NLS ($\beta=0$) and examine how changing β affects collision scenarios. In the case of two in-phase parallel solitons as input the colliding solitons behave periodically for a small nonlinear diffraction term [Fig. 3(a)], as in the case of the pure NLS. For larger β the colliding solitons merge into a single localized state, with breatherlike features [Fig. 3(b)]. For solitons moving in opposite directions, the colliding solitons feature similar behavior. One example, presented in Fig. 3(c), shows two solitons merging into a single localized state. Such inelastic effects often appear when dealing with a perturbed NLS; again, here it is caused purely by nonlinear diffraction. For two out-of-phase solitons the results show that the repulsive force between neighboring solitons is reduced with an increase of nonlinear diffraction [Fig. 3(d)].

Finally, we investigated soliton properties in the $(2+1)D$ model (two transversal dimensions and one propagation direction), but no stable solitons were found. The results show, however, that nonlinear diffraction modifies the critical power for collapse: positive (negative) β reduces (increases) the critical power. This means the nonlinear diffraction can slow down the collapse.

In most PhC structures the SC effect is relatively broadband. This means that the curvature changes slowly near the SC point. Therefore, to have a significant nonlinear diffraction effect one needs to design a PhC with a large curvature change, meaning a large β_1. At the SC frequency the nonlinear diffraction term would then be the main contribution that can interact with the normal nonlinear term.

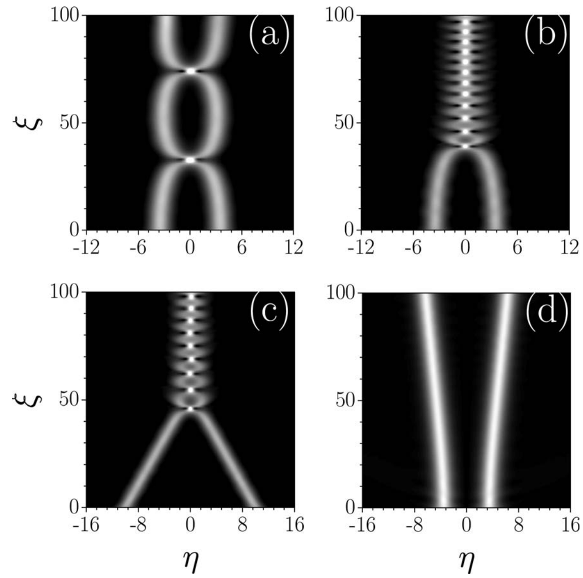

Fig. 3. Collision scenarios between solitons with $\alpha=\gamma=1$: two in-phase parallel solitons for $\beta=$ (a) 0.05 and (b) 0.15. (c) Two solitons moving in opposite directions with an angle ($\theta=0.2$) and $\beta=0.15$. (d) Two out-of-phase parallel solitons for $\beta=0.15$.

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