

## SEMI-ANALYTIC ANALYSIS OF COMPLEX PHOTONIC CRYSTAL STRUCTURES

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### SUMMARY

The transmission and reflection performance of complex photonic crystal structures coupled to dielectric waveguides are analyzed by means of a semi-analytic treatment based on previously derived closed-form expressions.

### KEYWORD

photonic crystals, eigenmode expansion

### ABSTRACT

### INTRODUCTION

Photonic crystals (PhCs), periodic structures with a period of the order of the wavelength of light, have been the subject of an increasing research effort due to their ability for controlling the flow of light. However, efficient coupling is a key feature to ensure their optimum performance. In recent years, a large variety of different techniques have been proposed and evaluated by means of simulation to improve the coupling efficiency. However, only a few works have been focused on the modeling of the interface between PhC circuits and external media (fiber or dielectric waveguides) [1-5]. The modeling of PhC circuits with efficient and accurate approaches may significantly reduce the computation time, which is usually very long in conventional numerical methods such as the finite-difference time-domain (FDTD) method [6]. In this paper, previously derived closed-form expressions for the transmission and reflection at an interface between a dielectric waveguide and a semi-infinite PhC waveguide are used to analyze more complex structures by means of a semi-analytic treatment.

### SEMI-ANAYTIC APPROACH

The proposed approach is valid for any kind of complex structure as long as the input medium has an index profile invariant along the propagation direction and the output medium is semi-infinitely periodic along the propagation direction or vice versa. The transmission and reflection matrices, derived based on an eigenmode expansion technique and a Bloch basis [5,7], are defined as

$$T = F_+^{-1} (I - R_{21} B_+ F_+^{-1})^{-1} T_{12} \quad (1)$$

$$R_{IN} = R_{12} + T_{21} B_+ T \quad (2)$$

$$R_{OUT} = -(B_- - R_{21} F_-)^{-1} (B_+ - R_{21} F_+) \quad (3)$$

where  $F_+$  and  $B_+$  are the forward and backward components of the forward propagating Bloch modes while  $F_-$  and  $B_-$  are the forward and backward components of the backward propagating Bloch modes. The transmission and reflection matrices  $T_{12}$ ,  $T_{21}$ ,  $R_{12}$  and  $R_{21}$  depend on the structure placed between the invariant and semi-infinite periodic media and must be calculated with a numerical tool giving thus the semi-analytic character to the proposed approach. In this work, the forward and backward components of the Bloch modes as well as  $T_{12}$ ,  $T_{21}$ ,  $R_{12}$  and  $R_{21}$  matrixes were calculated with a frequency-domain model based on a vectorial eigenmode

expansion technique, known as CAMFR and freely available from the Internet [8]. Fig. 1(a) shows an example of structure that can be analyzed.  $T_{12}$ ,  $T_{21}$ ,  $R_{12}$  and  $R_{21}$  are calculated for medium I-II considering that the input interface is a dielectric waveguide and the output interface is the layer with an invariant index profile that depends on the cut position in the PhC waveguide, as depicted in the middle part of Fig. 1(a). The transmission,  $T$ , and reflection,  $R_{IN}$ , matrices describe the coupling for light propagating from the invariant medium (medium I in Fig. 1(a)) into the periodic structure (medium II in Fig. 1(a)) while the reflection matrix,  $R_{OUT}$ , describes the coupling for light propagating from the periodic medium into the invariant medium. In the latter case, the transmission matrix can be simply obtained by the transpose of  $T$  due to the reciprocity theorem.

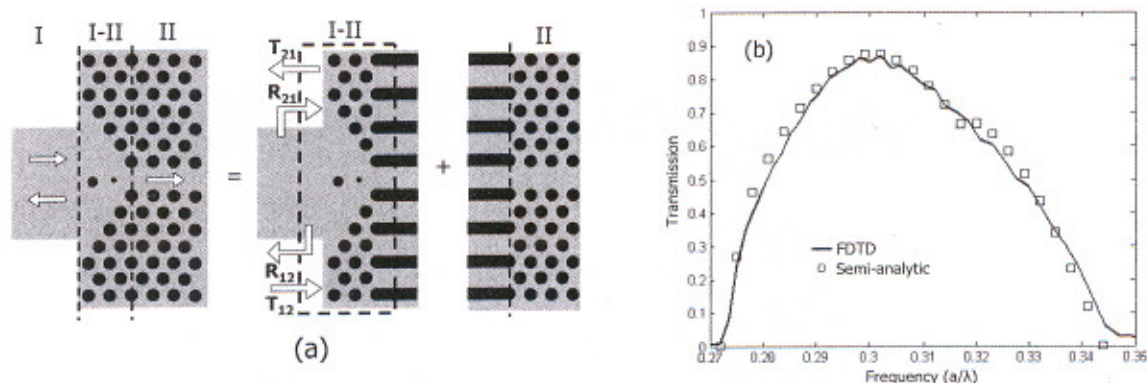


Fig. 1. (a) First analyzed structure formed by a dielectric waveguide coupled to a semi-infinite photonic crystal (PhC) waveguide by using an especially designed two-defect configuration placed within a PhC taper to improve the coupling efficiency. (b) Transmitted power as a function of the normalized frequency for the structure shown in (a).

#### COUPLING INTO LINE-DEFECT PHOTONIC CRYSTAL WAGUIDES

The analytic expressions have been firstly used to analyze the structure shown in the left part of Fig. 1(a) considering a  $3\mu\text{m}$ -wide dielectric waveguide of silica ( $n=1.45$ ) surrounded by an air cladding and a PhC formed by a two-dimensional triangular lattice of dielectric rods of Silicon ( $n=3.45$ ) embedded in silica. The radius of the rods is  $R=0.2a$  where  $a$  is the lattice constant. The structure shown in Fig. 1(a) was previously analyzed by means of FDTD simulations in Ref. 9. The transmitted power as a function of the normalized frequency is shown in Fig. 1(b). Semi-analytic results are compared to FDTD simulations. In the latter, the fundamental mode of the dielectric waveguide was excited with a monochromatic continuous wave (CW) with normalized power. The transmitted power was then calculated by integrating the power flux inside the PhC waveguide. It can be seen that there is a very good agreement between semi-analytic results and FDTD simulations.

Fig. 2(a) shows the spectrum of the reflected power into the dielectric waveguide. Semi-analytic results were calculated by using (2). It is important to notice that the  $3\mu\text{m}$ -wide dielectric waveguide is multimode for the frequency range of interest. The waveguide supports two even guided modes below the frequency of  $0.3(a/\lambda)$  but three even guided modes above this frequency. In FDTD, the reflected power was obtained by integrating the power flux along the dielectric waveguide at a certain distance before the position of the light source. Therefore, the power carried by each of the guided modes can not be separately calculated and only the total power is obtained. However, the analytic expressions allow calculating the reflection into each one of the guided modes. In Fig. 2(a), it can be seen that the power is not only reflected into the fundamental mode but it is spread into the different guided modes that the dielectric waveguide supports. Furthermore, it is also shown that the sum of the power carried by each of the guided modes is in agreement with the reflected power calculated by FDTD, which demonstrates the validity of the analytic expressions.

The reflection into the PhC waveguide when the light propagates from the PhC to the dielectric waveguide has also been analyzed by using (3). Fig. 2(b) shows the reflected power as a function of the normalized frequency. In this case, FDTD simulations results by using a monochromatic CW were not accurate because the fundamental guided Bloch mode could not be excited within the PhC waveguide. However, semi-analytic results can still be validated by calculating by means of FDTD the transmitted power of a PhC waveguide of finite length coupled to input and output dielectric waveguides using the optimized PhC taper. The structure is the same of that considered in Ref. 9. Semi-analytic results (solid line) show that there is a frequency range between

$0.285(a/\lambda)$  and  $0.3(a/\lambda)$  where the reflection is almost negligible. Therefore, the Fabry-Perot resonances do not appear in the transmission spectra calculated with FDTD (dashed line) at these frequencies.

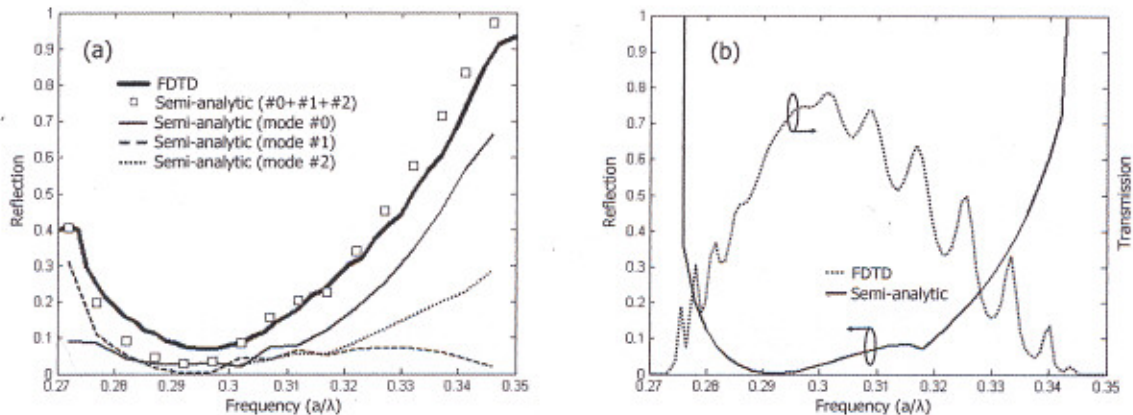


Fig. 2. (a) Reflected power into the dielectric waveguide as a function of the normalized frequency for the structure depicted in Fig. 1(a). Semi-analytic results are shown for each of the guided modes in the dielectric waveguide as well as for the total sum of the power, which is compared to that calculated by FDTD simulations. (b) Reflected power into the PhC waveguide as a function of the normalized frequency for the structure shown in Fig. 1(a) considering that the light propagates from the PhC into the dielectric waveguide. The transmitted power is also shown for the same structure used to couple light into and out of a PhC waveguide of finite length.

#### COUPLING INTO COUPLED-CAVITY WAVEGUIDES

The proposed approach can also be used to analyze the transmission and reflection properties of more complex structures, as the one shown in Fig. 3(a), which consist of a coupled-cavity waveguide (CCW) coupled to a conventional PhC waveguide by using an adiabatic taper based on progressively varying the radii of the spacing defects between the cavities that form the CCW. The details of the structure can be found in Ref. 10. In this case, we are interested in calculating the reflection into the CCW when the light propagates from the CCW to the PhC waveguide through the adiabatic taper. In principle, both media are periodic so that the analytic expressions are no longer valid. However, a simple trick can allow the use of the analytic expressions. Fig. 3(a) shows the analyzed structure. The dashed square corresponds to medium I-II in which  $T_{21}$ ,  $R_{12}$  and  $R_{21}$  have to be numerically calculated. The PhC waveguide has been butt coupled to a  $0.5\mu\text{m}$ -wide dielectric waveguide by conveniently choosing the cut position to achieve negligible reflection in the whole bandwidth of the CCW [7]. This is possible since the bandwidth of the PhC waveguide is much broader than that of the CCW. Therefore, the reflection calculated using (3) will be only the one that is originated from the adiabatic taper. Once more the validity of the semi-analytic results has been demonstrated by comparing with the transmitted power obtained by means of FDTD for a CCW of finite length coupled to the input and output PhC waveguide by using the adiabatic taper [10]. The adiabatic taper considered is formed by 9 intermediate rods with a linear variation of their radius. In Fig. 3(b), it can be seen that the Fabry-Perot resonances only appear in the transmission spectrum, calculated with FDTD, at the frequencies in which the reflection, calculated by means of the analytic expressions, increases.

#### CONCLUSION

In summary, an efficient approach has been proposed for a semi-analytic treatment of complex PhC structures. The computation time may be significantly reduced with respect to other conventional numerical methods, such as FDTD, since the full structure need not be simulated. Furthermore, it is also possible to analyze the reflection into PhC circuits, which can be very useful when testing novel designs and studying the influence of different parameters.

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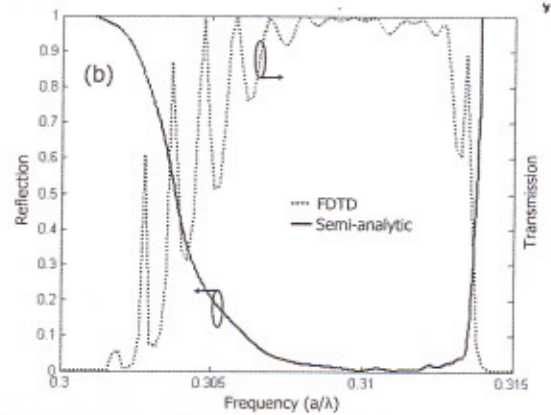
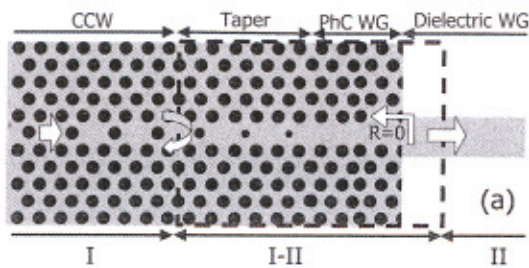


Fig. 3. (a) Second analyzed structure formed by a coupled-cavity waveguide (CCW) coupled to a conventional PhC waveguide by using an adiabatic taper. The PhC waveguide is butt coupled to a  $0.5\mu\text{m}$ -wide dielectric waveguide by conveniently choosing the cut position to achieve negligible reflection back to the CCW. (b) Reflected power into the CCW as a function of the normalized frequency considering an adiabatic taper formed by 9 intermediate rods with a linear variation of their radius. The transmitted power is also shown for the same structure used to couple light into and out a CCW of finite length.

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