On the Distinctive Features of Gain Coupled DFB Lasers and DFB Lasers with Second-Order Grating

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Abstract—**First, the two types of DFB lasers with gain coupling, true gain coupled lasers (with either a gain or loss grating) and second-order index coupled lasers are shown to be equivalent, mathematically and to some extent also physically. The operation of these DFB lasers, partly based on the overlap of the standing wave pattern in the optical power density with the periodic gain/loss variation, is addressed. It is further shown in detail how this special feature also leads to important modifications in the expressions for the modal loss or threshold gain, the threshold gain difference, the efficiency and the linewidth.**

I. Introduction

It has already been shown experimentally that gain coupled DFB lasers have several advantages in comparison with conventional index coupled DFB lasers [1]–[3]. One of the typical features of gain coupling, the large influence of the overlap between the standing wave patterns in both the optical power density and the gain or the loss [4], leads to a better mode discrimination and in some cases to a lower threshold gain. The better mode discrimination results, for as-cleaved gain coupled lasers, in a single-mode yield that is considerably higher than that of index coupled lasers and that makes AR-coatings no longer necessary. The higher yield has been demonstrated both theoretically [5] and experimentally [2]. Experimental single-mode yields as high as 95% have been reported. This figure has to be compared with the corresponding figure for index coupled lasers, which is below 50%. Other advantages of the introduction of gain coupling are the reduced reflection sensitivity and the reduced spatial hole burning [6].

The high yield is of great importance for advanced optical communication systems, which require stable single-mode lasers. In addition, the reduced spatial hole burning may result in a lower linewidth, a more uniform FM response (as a function of frequency and bias level and from one device to another) and a smaller harmonic distortion.

However, the influence of the standing wave patterns in power density and gain complicates the theoretical analysis of gain coupled lasers considerably. Although it has not yet been pointed out, several quantities such as the modal loss, the efficiency, and the linewidth are modified by the overlap of both standing wave patterns. The exploration of these modifications for gain coupled lasers is the subject of this paper.

Since gain coupled lasers show some similarity with second-order index coupled lasers (they are both described by a complex coupling coefficient), we start by pointing out the exact relations between both types of DFB lasers. It is shown that a second-order index coupled laser is mathematically and to some extent also physically equivalent with a loss coupled laser and therefore its complex coupling also leads to a larger mode discrimination as in gain coupled lasers. A second paragraph discusses the extra losses that have to be introduced to obtain significant gain coupling. Both gain and loss gratings are considered and in both cases the duty cycle of the grating can be identified as an important design parameter. The third paragraph then finally shows how to calculate threshold gain, threshold gain difference, efficiency, and linewidth. Thereby the influence of the standing wave pattern on these quantities can be seen. Some differences between lasers with gain grating and lasers with loss grating are pointed out here. In lasers with a gain grating, the grating causes a correlation between forward and backward propagating spontaneous emissions and gives rise to a modified stimulated emission rate in the carrier rate equation. This is not in the case in lasers with a loss grating.

II. Relation Between Gain Coupled and Second-Order DFB Lasers with Radiation Losses

Two types of DFB lasers described by a "complex" coupling coefficient are known: gain coupled DFB lasers with a periodically varying gain or absorption coefficient and index coupled DFB lasers with a second-order grating where the excitation of radiation modes results in losses that can also be described by a periodically varying absorption or rather loss coefficient α_{rad}. While the similarity between both laser types with complex coupling can already be understood from the above, simple picture, the more subtle differences between the different laser types require more attention.

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The basic properties of second-order DFB lasers have been established in the literature [7], [8]. The coupling coefficients are given by [7]:

$$\kappa_{FB} = \kappa_{BF} = \kappa_{\text{index}} + j\kappa_{\text{rad}} \tag{1}$$

with κ_{index} being a measure for the second-order index coupling. An expression for κ_{rad} can be found in the literature [7], [8]. $j\kappa_{\text{rad}}$ satisfies the mathematical relation characteristic for a pure gain grating ($\kappa_{BF} = -\kappa_{FB}^*$) and the second-order index coupled DFB laser seems mathematically equivalent to the partly gain coupled laser.

The constant term in (1) implies that the average internal loss α_{int} increases and as such the second-order index coupled laser becomes closely related to the partly gain coupled laser with a loss grating [2]. Such a grating is obtained by periodically increasing the absorption and thus also implies an increase of the average loss.

One difference between a second-order laser and a loss coupled laser lies in the relation between coupling coefficient and threshold gain. For the case of radiation losses one easily finds a loss increase given by $2\kappa_{\text{rad}}$. The modal gain 2α is then given by:

$$2\alpha = \Gamma g_o - \alpha_{int} - 2\kappa_{\text{rad}} \tag{2}$$

with g_o is the material gain, Γ is the confinement factor of the active layer, and α_{int} is the internal loss of the unperturbed waveguide. The relation $2\alpha = \Gamma_{ac} g_{mat} - \alpha_{int}$ is the well-known relation for an index coupled DFB laser. The extra term in (1), that is—$2\kappa_{\text{rad}}$, describes the losses due to radiation. The extra loss associated with a loss grating is again proportional with the imaginary part of the coupling coefficient κ_{gain}, but the ratio between both depends on the specific form of the grating. This extra loss is discussed in more detail in the next section. Another difference can be found in the value of the coupling coefficients. The value of κ_{rad} is typically limited to 5–10 cm^{-1}, while loss gratings allow gain coupling coefficients as high as 50 cm^{-1}.

III. Extra Losses of Loss and Gain Gratings

Before looking closer at the details of gain and loss gratings, we first want to emphasize that the detailed behavior of first (second-) order DFB lasers in general only depends on the zeroth and first order (and second order for second-order lasers) harmonics of the grating shape, irrespective of the coupling mechanism. Further details of the grating shape have therefore no influence whatsoever.

True gain gratings are not accompanied by extra losses. Though, we want to remind the reader that the gain coupling is usually proportional with the threshold gain, which decreases with increasing gain coupling. Strong gain coupling can therefore only be achieved by increasing the threshold gain via an increase of the internal loss or absorption. Furthermore, in the case of gain coupling we will use the notation Γg_o for the average gain over the periodic variation. With:

$$\Gamma g = (\Gamma g)_{min} + \Delta(\Gamma g);\ \Delta(\Gamma g) \geq 0 \tag{3}$$

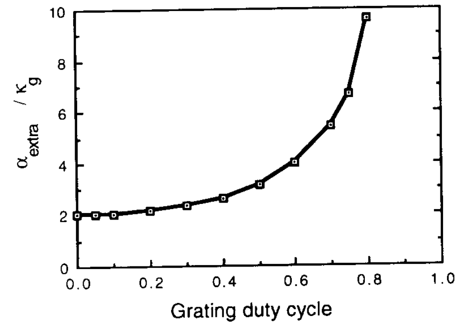

Fig. 1. Extra losses α_{extra} of GC DFB lasers with a loss grating function of the grating duty cycles.

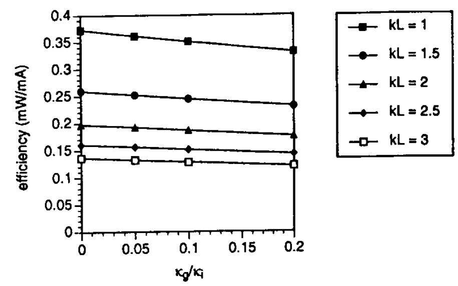

Fig. 2. Efficiency of PGC DFB lasers with a loss grating vs. κ_g/κ_i for different values of $\kappa_i L$ (L = 400 μm, $\alpha_{\text{int},0}$ = 40 cm^{-1}).

one has:

$$\Gamma g_o = (\Gamma g)_{min} + \frac{1}{\Lambda}\int_0^\Lambda \Delta(\Gamma g)\, dz \tag{4}$$

e.g., for a rectangular grating with duty cycle Λ_l/Λ:

$$\begin{aligned}\Gamma g_o &= (\Gamma g)_{min} + \frac{\Lambda_l}{\Lambda}\{(\Gamma g)_{max} - (\Gamma g)_{min}\}\\ &= (\Gamma g)_{min} + \frac{\Lambda_l}{\Lambda}\frac{2\pi}{\sin\left(\dfrac{\pi\Lambda_l}{\Lambda}\right)}\kappa_{\text{gain}}.\end{aligned} \tag{5}$$

We remind the reader that in the case of a corrugated active layer, gain coupling results from the periodic variation of the confinement factor. Periodic variations of the carrier density and the gain are washed out by the carrier diffusion (with a diffusion length that is typically one order of magnitude larger than the corrugation period).

Loss gratings are different because they cannot be fabricated unless if extra losses are introduced. The extra losses can be calculated in a similar way. Denoting by α_{int} the minimum internal loss along the grating, which now also corresponds with the loss in the absence of the grating, one finds:

$$\alpha_{\text{extra}} = \frac{1}{\Lambda}\int_0^\Lambda \Delta\alpha(z)\, dz \tag{6}$$

where $\Delta\alpha$ is the loss felt by the waveguide mode (= $\Gamma_{grat}\Delta\alpha_{mat}$), which depends on the confinement factor of

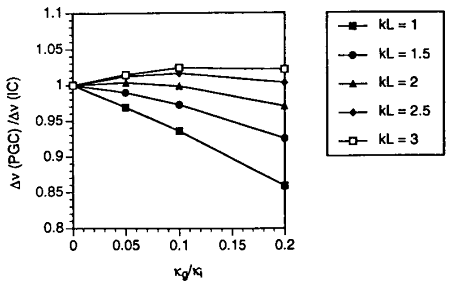

Fig. 3. Linewidth at 1 mW output power of PGC DFB lasers with a loss grating vs. κ_g/κ_i for different values of $\kappa_i L$ ($L = 400\ \mu$m, $\alpha_{\text{int},0} = 40$ cm^{-1}), relative to the linewidth of the index coupled laser with identical $\kappa_i L$.

the grating and on the variation of the material loss. (Figs. 1 and 2) For a rectangular grating one finds again:

$$\alpha_{\text{extra}} = \frac{\Lambda_l}{\Lambda} \frac{2\pi}{\sin\left(\frac{\pi\Lambda_l}{\Lambda}\right)} \kappa_{\text{gain}}. \tag{7}$$

The extra losses are shown in Fig. 3 as a function of the duty cycle. The minimum value $\alpha_{\text{extra}} = 2\kappa_{\text{gain}}$ can only be obtained with rectangular or triangular gratings with very small duty cycle. A grating with a sine form, e.g., gives an extra loss equal to $4\kappa_{\text{gain}}$. Remarkable is that the radiation at a second-order grating also seems to induce a loss grating with small duty cycle, independent of the shape of the second-order grating.

IV. The Standing Wave Pattern and its Influence on Threshold Gain, Threshold Gain Difference, Efficiency, and Linewidth

In the following, we consider a general DFB laser with complex coupling coefficients and with both index and gain coupling. The gain coupling can have its origin in a gain or loss grating or in radiation loss for second-order lasers. The general form of the coupled-mode equations is:

$$\frac{\partial R^+}{\partial z} + (j\delta - \alpha)R^+ = (\kappa_{\text{gain}} + \kappa_{\text{index}})R^- \tag{8}$$

$$\frac{\partial R^-}{\partial z} - (j\delta - \alpha)R^- = (-\kappa_{\text{gain}}^* + \kappa_{\text{index}}^*)R^+ \tag{9}$$

with κ_{gain} and κ_{index} complex numbers of which the phase depends on the phase of the grating. For simplicity, we will assume $\kappa_{\text{gain}} = \kappa_g$ and $\kappa_{\text{index}} = j\kappa_j$ with κ_g and κ_j both real numbers. This corresponds with the case where the gain and index coupling are provided by only one grating (or more generally with a phase difference of 0 or π between the gain and the index grating) which has a phase 0 or π at $z = 0$. From (8) and (9), one can easily derive equations for the amplitudes of R^+ and R^- (see the Appendix) and from the last equations one can derive the equation for the number of photons I inside the cavity:

$$I = \frac{1}{\hbar\omega v_g} \int_0^L \{|R^+|^2 + |R^-|^2\}\, dz \tag{10}$$

One finds

$$2\alpha I + \kappa_g f_{st} I = 2\alpha_{\text{end}} I;$$

with

$$2\alpha_{\text{end}} I = \frac{1}{\hbar\omega_{v_g}} \{(|R^+|^2 - |R^-|^2)_{z=L} + (|R^-|^2 - |R^+|^2)_{z=0}\}$$

and

$$f_{st} = \frac{\int_0^L \text{Re}\,(2R^+R^{-*})\, dz}{\int_0^L (|R^+|^2 + |R^-|^2)\, dz} : \text{the standing wave factor.} \tag{11}$$

The threshold gain is thus determined by the following condition:

$$2\alpha = \Gamma g_0 - \alpha_{\text{int}} = 2\alpha_{\text{end}} - 2\kappa_g f_{st} \tag{12}$$

with α_{int} possibly including extra losses (as for a loss grating and radiation or intentionally introduced to make strong gain coupling possible). κ_g and f_{st} can both be positive or negative. The sign of κ_g can be either positive or negative. A positive (negative) sign corresponds with gain (loss) at $z = 0$. Lasing however will occur in the mode for which the threshold gain is minimum and for which $\kappa_g f_{st}$ is positive and maximum.

It is shown in the Appendix how, for AR-coated DFB lasers with complex coupling coefficient, f_{st} can be expressed analytically in terms of δ and κ_i:

$$f_{st} = \frac{\kappa_i}{\delta}. \tag{13}$$

The derivation of this relation also be generalized to lasers with reflecting facets and one would obtain an extra dependence on α_{end} and on the amplitudes and phases of the facet reflectivities. It must be emphasized however that (13) provides an exact expression for the standing wave factor of AR-coated partly gain coupled lasers.

Expression (13), when inserted in (12) explains the extra mode discrimination that exists in partly gain coupled and second-order lasers. If, for simplicity, we assume that the two main modes are located on either side of the Bragg wavelength with $\delta = \pm\kappa_i$ (an approximation valid for small κ_g), we readily find the following threshold gain difference:

$$2\Delta\alpha L \approx \frac{4\kappa_g\kappa_i L}{\kappa_i} = 4\kappa_g L \tag{14}$$

This equation implies that a threshold gain difference $2\Delta\alpha L$ of 0.2, which normally ensures stable single mode behavior, is, for a typical laser length of 400 μm, already obtained for a value of $\kappa_g = 1.25$ cm^{-1}, a value that is achieved even with second-order gratings. The formula (14) was found earlier by Makino and Glinksi [7] via a perturbation approach. Here we have used an alternative approximation and (13) allows to derive an exact formula if the δ of both main and side mode can be found. The influence of the $2\kappa_g f_{st}$-factor is also the reason for the suppression of the Fabry–Perot resonances on one side of the Bragg-wavelength in the spectrum of lasers with complex coupling.

The threshold gain of the lasing mode can be approximated from:

$$\Gamma g_0 - \alpha_{int} = 2\alpha_{\text{end}} - 2l\kappa_g f_{st} l \approx 2\alpha_{\text{end}} - 2l\kappa_g l. \quad (15)$$

The interpretation of the standing wave overlap $\kappa_g f_{st}$ is important in the calculation of the efficiency. A simple expression for the efficiency is found if one makes a clear distinction between true gain gratings and loss or second-order gratings. In the first case, one can define the efficiency as the ratio of the facet loss to the total loss. The grating is not affecting the loss and one obtains:

$$\eta = \frac{2\alpha_{\text{end}}}{2\alpha_{\text{end}} + \alpha_{int}} \left(= \frac{2\alpha_{\text{end}}}{\Gamma g_0 + 2\kappa_g f_{st}} \right). \quad (16)$$

In the case of loss gratings or second-order gratings, the alternative definition of the efficiency as the ratio of the facet loss rate to the stimulated emission rate is more appropriate. Now, the stimulated emission rate is not affected by the grating and one finds:

$$\eta = \frac{2\alpha_{\text{end}}}{\Gamma g_0} \left(= \frac{2\alpha_{\text{end}}}{2\alpha_{\text{end}} + \alpha_{int} - 2\kappa_g f_{st}} \right). \quad (17)$$

At this point one might wonder about the difference between gain and loss gratings. After all, can a loss grating not be considered as a gain grating shifted over half a period and why then is there no general formula for the efficiency? The answer lies in the carrier rate equation which must also be included when calculating the external, measurable efficiency. This rate equation contains the stimulated emission rate, which for gain gratings (but not for loss gratings) includes a contribution from the overlap of the fields' standing wave pattern with the periodic gain variation. In conclusion, for true gain gratings one must modify the stimulated emission rate in the carrier rate equation to:

$$R_{stim} = v_g(\Lambda g_0 + 2\kappa_g f_{st}) I \quad (18)$$

or, formulated alternatively, one should make a distinction between carrier dependent gain/loss variations and carrier independent gain/loss variations, rather than between gain and loss gratings.

The influence of the extra loss on the efficiency of a loss grating is, for a grating with small duty cycle, largely compensated by an opposite influence of the overlap term $2\kappa_g f_{st}$. This influence is illustrated in Fig. 2 for partly gain coupled AR-coated lasers with different values of $\kappa_i L$ and $\kappa_g L$. An extra loss $\alpha_{\text{extra}} = 2\kappa_g$ has been assumed for the calculations. Also the influence of κ_g on α_{end} has been taken into account and one sees that this influence remains limited. It can thus be concluded that the efficiency of partly gain coupled lasers with both gain and loss gratings is only a little smaller than the efficiency of the corresponding index coupled lasers.

In analogy to Fabry–Perot laser diodes, the linewidth $\Delta\nu$ of DFB lasers can be expressed approximately as (i.e., when spatial hole burning effects and other nonlinearities are neglected) [11]:

$$\Delta\nu = \frac{R_{sp}}{4\pi I}(1 + \alpha_{\text{eff}}^2) \quad (19)$$

where R_{sp} is the spontaneous emission rate and α_{eff} is the effective linewidth enhancement factor [12]. Following the approach in [11] we can write (19) as:

$$\Delta\nu P_0 = \frac{v_g^2 h\nu n_{sp}}{8\pi} \Gamma g_0 2\alpha_{g\,\text{end}}(1 + \alpha_{\text{eff}}^2) \quad (20)$$

where P_0 is the output power per facet, Γg_0 is the confinement factor of the active layer times the intensity material gain, v_g is the group velocity of light, $h\nu$ is the photon energy, and n_{sp} is the spontaneous emission factor.

For the case of gain coupling (i.e., a corrugated active layer), one should also take into account the dependence of the coupling coefficient on the carrier density, resulting in fluctuations of the coupling coefficient as a result of spontaneous emission. The inclusion of this dependency in a rate equation analysis is straightforward, but we will not discuss it here. This influence can, for traveling wave models such as CLADISS [12], be taken into account by assuming the proper carrier density dependence of the coupling coefficient. In addition, the moments of the Langevin functions appearing in the traveling wave equations:

$$\frac{\partial R^+}{\partial z} + \frac{1}{v_g}\frac{\partial R^+}{\partial t} + (j\delta - \alpha)R^+ = (\kappa_g + j\kappa_i)R^- + F^+(z, t) \quad (21)$$

$$-\frac{\partial R^-}{\partial z} + \frac{1}{v_g}\frac{\partial R^-}{\partial t} + (j\delta - \alpha)R^- = (\kappa_g + j\kappa_i)R^+ + F^-(z, t) \quad (22)$$

must be modified. The Langevin functions F^+ and F^- become correlated for a corrugated active layer (i.e., forward and backward propagating spontaneous emissions become correlated) and in this case (and only in this case):

$$\langle F^+(z, t)F^{+*}(z', t')\rangle = \langle F^-(z, t)F^{-*}(z', t')\rangle = \hbar\omega\Gamma g_0 n_{sp}\delta(z - z')\delta(t - t') \quad (23)$$

$$\langle F^+(z, t)F^{-*}(z', t')\rangle = \langle F^-(z, t)F^{+*}(z', t')\rangle = 2\hbar\omega n_{sp}\kappa_g\delta(z - z')\delta(t - t') \quad (24)$$

The expressions (23) are of course identical to the expressions that hold for index or loss coupling. These expressions can be derived from the Helmholtz equation for the electric field by integration over one cavity roundtrip time and over one grating period (and under the assumption that the normalization of the waveguide mode is such that the optical power is given by $R^+R^{+*} + R^-R^{-*}$) (A rigorous derivation of (23) and (24) is given in [13].) A grating with zero phase at $z = 0$ has been assumed in the derivation of (24).

Coming back to the linewidth we can write (20) for partly loss coupled lasers as:

$$\Delta\nu = \Delta\nu' \Delta\nu_{\text{norm}} \tag{25}$$

where $\Delta\nu'$ is given by

$$\Delta\nu' = \frac{v_g^2 h\nu n_{sp}}{P_0 L^2 8\pi} (1 + \alpha_{\text{eff}}^2) \tag{26}$$

and the normalized linewidth $\Delta\nu_{\text{norm}}$ is given by

$$\begin{aligned} \Delta\nu_{\text{norm}} &= \Gamma g_0 L 2\alpha_{\text{end}} L = 2\alpha_{\text{end}} L (\alpha_{int} + 2\alpha_{\text{end}} \\ &\quad - 2l\kappa_g f_{st} l) L \\ &\approx 2\alpha_{\text{end}} L (\alpha_{int} + 2\alpha_{\text{end}} - 2l\kappa_g l) L. \end{aligned} \tag{27}$$

For the laser parameters in (23) and (24) we take typical values for InGaAsP/InP DFB lasers as given in [6]:

$$v_g = c/4.33 \qquad h\nu = 0.8 \text{ eV} \qquad n_{sp} = 2.7$$
$$\alpha_o = 45/\text{cm} \qquad \alpha_{\text{eff}} = 5.4 \qquad L = 300\ \mu\text{m}$$

where c is the speed of light. This gives for $\Delta\nu'$ at an output power of P_o of 3 mW:

$$\Delta\nu' = 7.4 \text{ MHz}. \tag{28}$$

Notice that the normalized linewidth $\Delta\nu_{\text{norm}}$ for DFB lasers with a pure loss grating [(27)] differs from the corresponding expression for first-order index coupled lasers by the presence of $|\kappa_g f_{st}|$ ($\approx |\kappa_g|$ for ar-coated lasers) and by the possible presence of extra losses (included in α_{int}). The expression for $\Delta\nu'$ on the other hand is identical for index coupled lasers and lasers with complex coupling. Since weak or moderate gain coupling requires no extra loss, (27) implies that a linewidth reduction can be obtained by introducing small or moderate gain coupling. Since also the extra loss, needed to obtain loss coupling or strong gain coupling can be limited to about $2|\kappa_g|$ (e.g., by using a rectangular grating with small duty cycle), it can be argued that even in that case similar values for the linewidth as with index coupled lasers can be achieved. Because of the large threshold gain difference of GC DFB lasers, combined with low spatial hole burning very strong lasers should be possible. Hence GC DFB laser with loss grating have a potential advantage over IC DFB lasers. Indeed, experimentally a linewidth as low as 1.6 MHz for GC DFB lasers with loss grating has been shown [4].

V. Conclusions

First we have shown that a second-order index coupled DFB laser is equivalent, both mathematically and physically with a partly gain coupled DFB laser with a loss grating. This equivalence is a result of the periodically varying loss coefficient with the radiation loss, the period of this loss coefficient being half the period of the second-order grating. As in lasers with a real loss grating, the radiation loss also implies additional constant losses which are proportional with the imaginary part of the coupling coefficient. The proportionality factor depends on the grating shape for real loss gratings, but it is always equal to 2 for second-order gratings because they lead to a perfect cosine variation of the loss coefficient.

Another major part of this paper has been concentrated on important quantities such as threshold gain, threshold gain difference, efficiency, and linewidth, and the influence of the standing wave pattern on these quantities for lasers with complex coupling coefficients. The expressions found for all these quantities have been obtained from first principles, i.e., the derivation of a rate equation for the photon number from the coupled mode equations. Threshold gain and threshold gain difference depend on the overlap of the fields' standing wave pattern with the periodic gain/loss variation. We have shown for the first time how the overlap integral can easily be calculated analytically for AR-coated lasers.

The calculation of the efficiency requires more attention and it is different for gain and loss gratings. In the case of a gain grating, the overlap of the fields' standing wave pattern with the periodic gain forms an essential part of the stimulated emission rate and it must be included in the carrier rate equation, while this overlap must be included in the total loss in the case of a loss grating.

Finally, the linewidth follows easily from the previous quantities. It has been argued that the linewidth of a loss coupled laser can be as small as for the index coupled DFB laser (e.g., for a sufficiently small duty cycle in the case of a rectangular loss grating). The linewidth of real gain coupled lasers is also depending on the carrier density dependence of the coupling coefficient and on a cross correlation between forward and backward propagating spontaneous emissions, which makes the evaluation in this case far more complex. Further work is needed here.

For both gratings, the presented basic ideas are of significant interest for the design of optimized gain coupled DFB lasers.

Appendix

With the notations $R^\pm = |R^\pm| \exp(j\varphi^\pm)$, one finds from (8) and (9):

$$\begin{aligned} \frac{\partial |R^+|}{\partial z} - \alpha |R^+| &= \kappa_g |R^-| \cos(\varphi^- - \varphi^+) - \kappa_i |R^-| \\ &\quad \cdot \sin(\varphi^- - \varphi^+) \end{aligned} \tag{A.1}$$

$$\begin{aligned} \frac{\partial |R^-|}{\partial z} + \alpha |R^-| &= -\kappa_g |R^+| \cos(\varphi^- - \varphi^+) - \kappa_i |R^+| \\ &\quad \cdot \sin(\varphi^- - \varphi^+) \end{aligned} \tag{A.2}$$

$$\begin{aligned} \frac{\partial \varphi^+}{\partial z} + \delta &= \frac{|R^-|}{|R^+|} \{\kappa_g \sin(\varphi^- - \varphi^+) - \kappa_i \\ &\quad \cdot \cos(\varphi^- - \varphi^+)\} \end{aligned} \tag{A.3}$$

$$\frac{\partial \varphi^-}{\partial z} - \delta = \frac{|R^+|}{|R^-|}\{\kappa_g \sin(\varphi^- - \varphi^+) - \kappa_i \cdot \cos(\varphi^- - \varphi^+)\}. \quad (A.4)$$

Multiplication of (A.1) with $2|R^+|$, of (A.2) with $2|R^-|$ and subtraction of both equations gives:

$$2\alpha(|R^+|^2 + |R^-|^2) + \frac{\partial}{\partial z}(|R^-|^2 + |R^+|^2) = -4\kappa_g|R^+||R^-|\cos(\varphi^- - \varphi^+). \quad (A.5)$$

Integration of this equation from $z = 0$ to $z = L$ then gives:

$$2\alpha\int_0^L (|R^+|^2 + |R^-|^2)\,dz + 4\kappa_g\int_0^L |R^+||R^-| \cdot \cos(\varphi^- - \varphi^+)\,dz = (|R^-|^2 - |R^+|^2)_{z=0} - (|R^-|^2 - |R^+|^2)_{z=L} \quad (A.6)$$

which finally results in (11).

Multiplication of (A.1) with $|R^-|$, of (A.2) with $|R^+|$ and summation of the two resulting equation gives:

$$\frac{\partial(|R^+||R^-|)}{\partial z} = \kappa_g\cos(\varphi^- - \varphi^+)(|R^-| - |R^+|) - \kappa_i\sin(\varphi^- - \varphi^+)(|R^-| + |R^+|). \quad (A.7)$$

An equation for $|R^-||R^+|\cos(\varphi^- - \varphi^+)$ can be derived as follows. Equation (A.4) minus (A.3) gives:

$$\frac{\partial(\varphi^- - \varphi^+)}{\partial z} - 2\delta = -\kappa_g\frac{|R^-|^2 - |R^+|^2}{|R^+||R^-|}\sin(\varphi^- - \varphi^+) - \kappa_i\frac{|R^-|^2 + |R^+|^2}{|R^+||R^-|}\cos(\varphi^- - \varphi^+) \quad (A.8)$$

and multiplication of (A.7) with $\sin(\varphi^- - \varphi^+)$, of (A.8) with $|R^+||R^-|\cos(\varphi^- - \varphi^+)$ and summation of both equations gives:

$$\frac{\partial}{\partial z}\{|R^+||R^-|\sin(\varphi^- - \varphi^+)\} - 2\delta|R^+||R^-|\cos(\varphi^- - \varphi^+) = -\kappa_i(|R^-|^2 + |R^+|^2). \quad (A.9)$$

After integrating from $z = 0$ to $z = L$ one finds for AR-coated lasers:

$$2\delta\int_0^L |R^+||R^-|\cos(\varphi^- - \varphi^+)\,dz = \kappa_i\int_0^L (|R^-|^2 + |R^+|^2)\,dz$$

or

$$f_{st} = \frac{\kappa_i}{\delta}. \quad (A.10)$$

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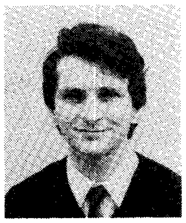

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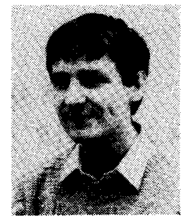

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